Random Walks on Products of Free Groups

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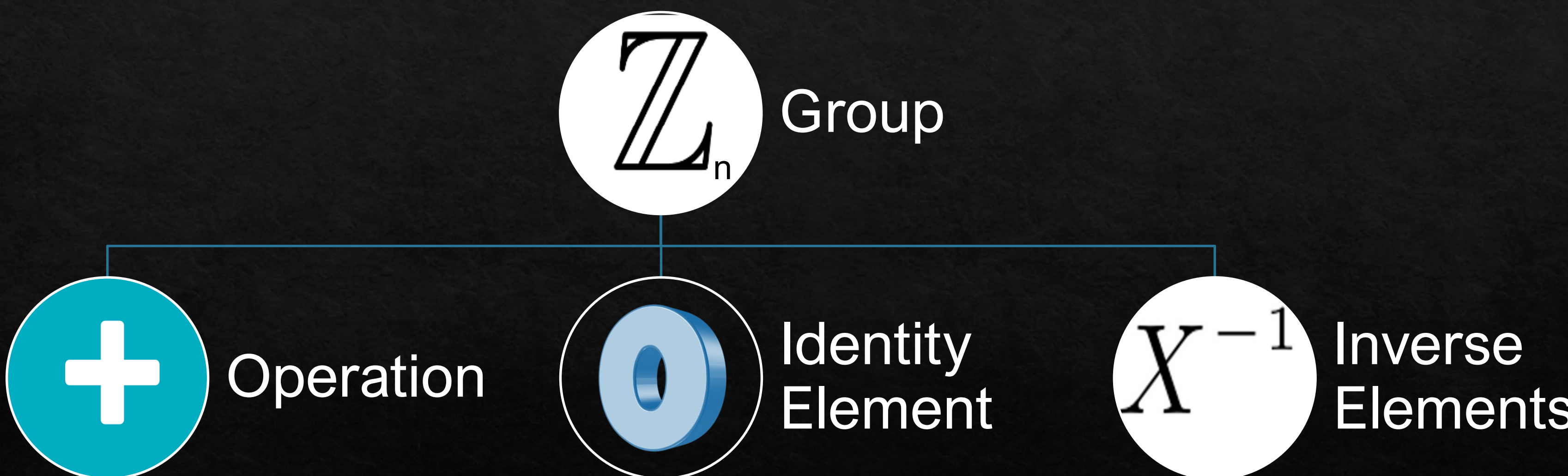
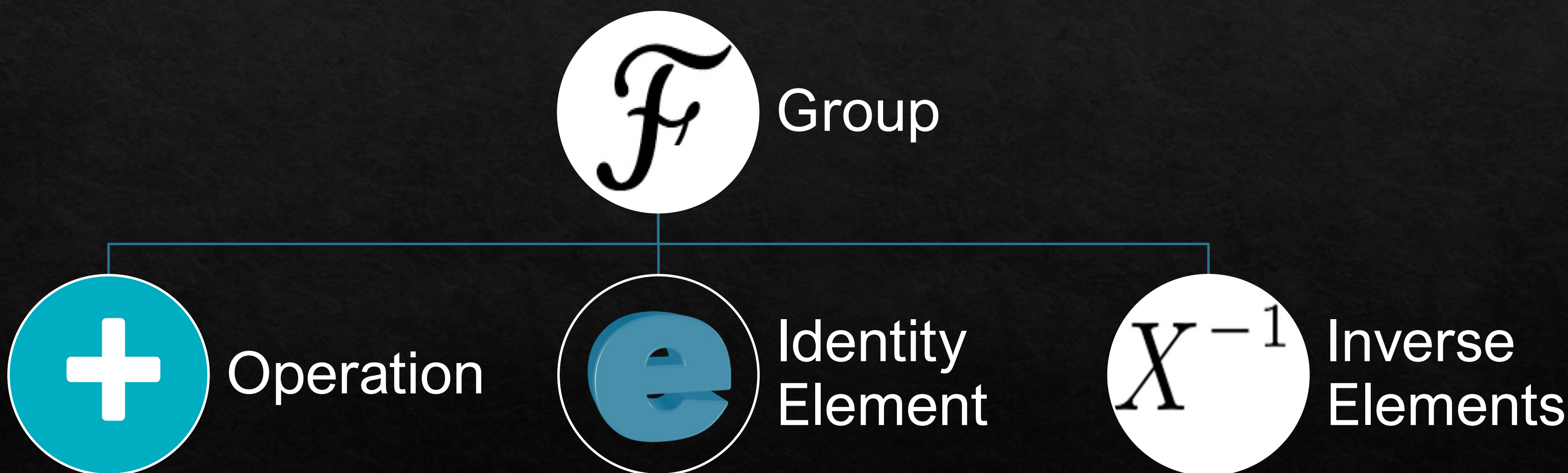
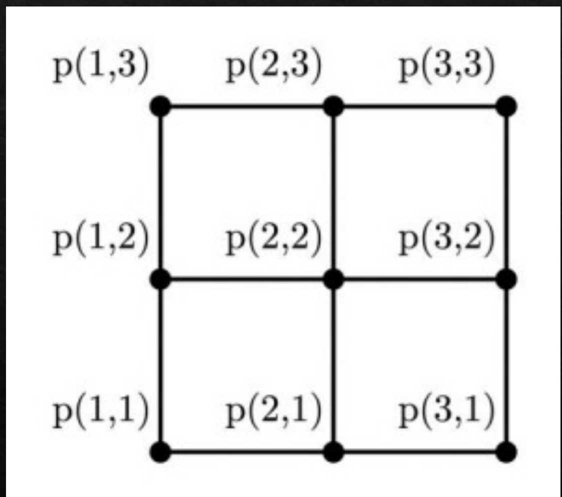
Abelian Group



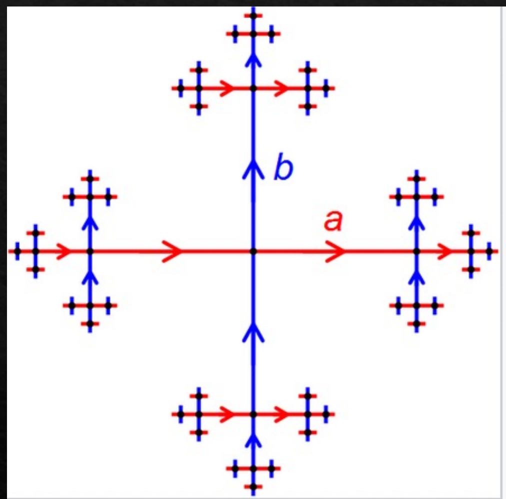
Free Group



Generators



This Figure was taken from Jarutatsanangkoon (2018).



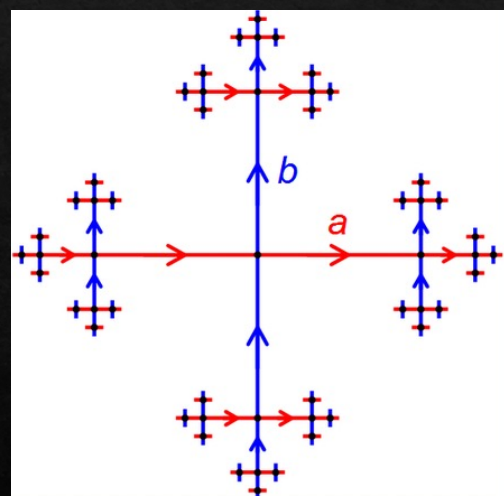
This Figure was taken from Wikipedia.

Notation

- μ(n)
  - The probability of moving in the direction n
- $\check{\mu}$(n)
  - The probability of moving in direction inverse to n so that $\check{\mu}(n)=\mu(n^{-1})$
- $\sigma = \mu * \check{\mu}$
  - σ contains the probability of staying in place
- The spectral norm (denoted $||\mu|| = ||\sigma||^{1/2}$)
  - The probability of returning to the identity after infinitely many steps
  - $||\sigma|| = \lim_{n\to\infty} \sqrt[n]{\sigma^{(n)}(e)}$

Leinert Property

- The only way to return is through backtracking



This Figure was taken from Wikipedia.

Goal

Methods

Results

Goal

Relax the backtracking behavior

Woess solves for the radius of convergence of G(z) Leinert Property and Recursion

$$G(z) = \sum_{n=0}^{\infty} \mu^{(n)}(e) z^{2n}$$

The spectral norm is the inverse of the radius of convergence of G(z)

Goal

An instance of this is the product of free groups.

Goal

Methods

Results

Methods

Random walks can be simulated with random unitary matrices

Goal

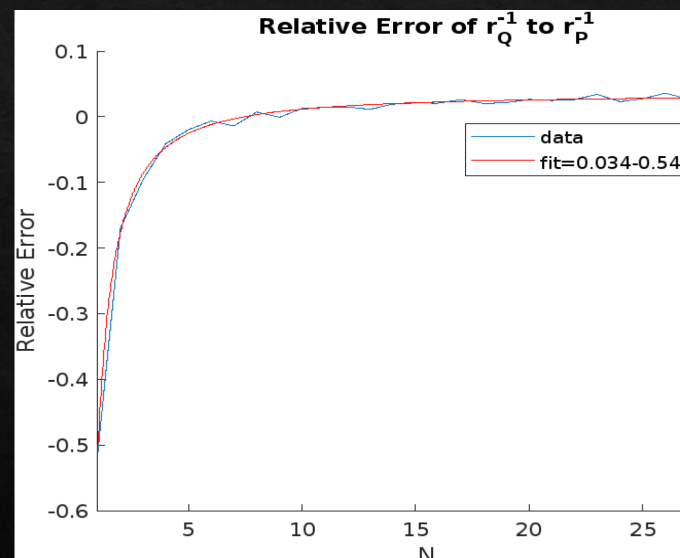
Methods

Results

Results

Upper Bound for bad walks

Found an equation Q(t,z) whose r^{-1} bounds the spectral norm



Goal

Consider Almost Leinert

Methods

Counting through Random Unitary Matrices

Results

~3% of relative error

Results have implications in various fields.

- The most prominent fields are:
  - Mathematics
    - Algebra, Linear Algebra, and Graph Theory
  - Quantum Mechanics
    - Electron behavior

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